Journal of Engineering Physics and Thermophysics, Vol. 77, No. 1, 2004

EULERIAN INSTABILITY OF CHARGED FIXED PLATES

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The instability of a thin elastic charged fixed plate has been considered. It has been proved that the threshold value of the square of the electric-field strength grows linearly with tension of the plate and its rigidity.

There is a wide class of elastic objects (rods, plates, shells) [1] that exhibit the phenomenon of longitudinal instability under the action of longitudinal compressive forces. Euler was the first to discover this phenomenon using the behavior of a rod in longitudinal compression as an example [2].

In the absence of transverse deflecting external forces, the rod remains rectilinear under the action of a longitudinal force. This state, however, corresponds to a stable equilibrium of the rod only as long as the compressive force remains smaller than a certain critical action. When $F < F_{cr}$, the rectilinear shape of the rod is stable to an arbitrary perturbation. In other words, if the rod is weakly bent under the influence of any small action, it will tend to return to the initial state in the case of cessation of this action.

On the contrary, when $F > F_{cr}$, the rectilinear shape corresponds to an unstable equilibrium. Even an infinitely small action (bending) would suffice to upset the equilibrium, which will result in a strong bending of the rod. It is clear that the compressed rod will be unable to exist in unbent form under these conditions at all.

It is obvious that such consideration is applicable to plates that are a limiting case of a rod with one transverse dimension substantially larger than the other.

Let us consider such instability for electrically charged plane plates. Indeed, in the case of a small (uniformly "smeared" over the plate) charge, the initial plane state is equilibrium for them; forces do not act on the plate's element in the transverse direction. However, when a certain critical charge density is exceeded, the bent surface is stable. Let us determine this critical value and the shape of the surface in the case of stability loss. To solve the problem we employ a system of notation analogous to that adopted in [2].

Let the plate be fixed along straight lines which are perpendicular to the plane (x, z) and whose projections onto the plane have coordinates $x = 0$, $z = 0$ and l ($x(z)$ is the deviation of the plate from the initial undeformed position as a function of the coordinate z; the deformation is assumed to be independent of the coordinate y).

Let the plate be electrically charged with a surface density σ. When the surface deviates from equilibrium by the quantity x, the normal force P acts (in the x direction) on the unit area; the value of the force is determined just in the same manner as the pressure on a charged surface (in investigating the Frenkel–Tonks instability [3]) if the perturbation of the surface is represented in the form $x = x_0 \exp(ikz)$; without loss of generality, we take $k > 0$:

$$P = Akx \,, \quad A = 4\pi\sigma^2 \,. \tag{1}$$

To determine the conditions of stability loss by the plane shape of the plate we use the equation of elasticity theory [2], describing the bending

$$EI \frac{d^4x}{dz^4} - T \frac{d^2x}{dz^2} - P = 0 \,, \tag{2}$$

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 1062-0125/04/7701-0216 ©2004 Plenum Publishing Corporation

where I is the moment of inertia per unit length along the y direction ($I = a^3/12$) and T is the force acting along the plate's edge in the z direction per unit length in the y direction: if $T > 0$, the force is produced by external tensile forces and conversely.

Substituting the expression for P from (1) into (2) and assuming x in the form of a wave exponent (see above), we obtain the characteristic equation for determination of k:

$$EIk^4 + Tk^2 - Ak = 0 \, . \tag{3}$$

The only nonzero real root determined from the cubic equation (after the removal of k in (3)) with the use of the Cardan formula [4] has the form

$$k = \sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} + \frac{q}{2}} - \sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} - \frac{q}{2}} \, , \tag{4}$$

where $p = T/EI$ and $q = A/EI$; the solution of (4) is written for the simplest case $p > 0$, i.e., for the case of forces extending the film (in the direction along the z axis).

The boundary conditions for $z = 0$ and $z = l$ are determined by the fixing employed. Let a hinging on both straight lines be performed. Then we have $x = d^2x/dx^2 = 0$ for $z = 0$ and $z = l$. The solution of Eq. (2), satisfying the above boundary conditions, has the form $x = C \sin (kz)$; here, $kl = \pi n$ (n is the integer) and C is the constant. The threshold of stability loss corresponds to $n = 1$, i.e., to the condition

$$kl = \pi \, , \tag{5}$$

where k is taken from Eq. (4).

After cubing and simple algebraic transformations, we determine the threshold value of the electric field ($E \sim \sigma$)

$$A = \frac{\pi}{l}\left(T + \frac{\pi^2}{l^2} El\right). \tag{6}$$

The result (6) is quite expected: the threshold value of the field (of its square, to be more precise) linearly grows with the tension of the shell and its rigidity.

Thus, the charged elastic plate loses its stable plane shape on attainment of a certain threshold surface electric charge; the instability is realized analogously to the Euler instability.

In closing, we call your attention to one interesting (from our viewpoint) circumstance. The fact is that the occurrence of the third-power equation is very characteristic of problems of stability loss in the elastic field. Thus, as has been shown in [5], it is precisely this circumstance that has led one to a fundamentally new result — duplication of the solution of the problem of bifurcation-type stability (these considerations have been presented in greater detail in [6]). We note that, judging from [7] (p. 65), the bifurcation character of stability disturbance is universal. The example presented at the beginning of the paper — the rod loses its stability, and it is unclear in what direction its end will be displaced — can be a good illustration of this circumstance.

NOTATION

a, plate thickness, m; E, elastic modulus, Pa; k, wave number, 1/m; l, plate length, m; σ, surface density of electric charges, C/m^2. Subscript: cr, critical.

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